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REPRESENTATION OF TWO-PHASE FLOWS BY VOLUME AVERAGING

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Abstract—We consider descriptors of gas–particle aggregates which represent space-averaged local properties of the aggregate. We show that such descriptors have undulations due to the finite size of the averaging volume, and derive an estimate for the bounds of the amplitude of the undulations. According to that estimate, one obtains reasonably accurate averages only if the averaging volume contains at least 60–150 particles. In terms of the size of the averaging volume, this limitation means that the diameter of the volume should equal at least 4–5 mean distances between particle centers. A consequence for the modeling of two-phase flows through tubes is that space-averaged descriptors cannot resolve the radical structure of such flows unless the mean particle distance is much smaller than one-tenth of the tube diameter. This condition excludes, for instance, the use of average descriptors for radial flow resolution of some interior ballistics flows, where the particulate phase consists of propellant grains.

1. INTRODUCTION

This paper deals with two-phase flows which consist of a gas–particle mixture. Such flows are described commonly by averages of the local properties of the gas and particles (Drew 1971; Gough 1974; Delhaye & Achard 1977; Nigmatulin 1979; Immich 1980; Gibeling *et al.* 1980; Celmiņš & Schmitt 1984; Dobran 1984). The flow descriptors can be averaged in various ways, e.g. over a time domain, over a spatial dimension etc. Comparing the different averaging methods, one finds that most useful in terms of general properties and applicability to unsteady flows, is volume averaging (Dobran 1984; Celmiņš & Schmitt 1984). Conceptually, volume averaging might be interpreted as a view of the flow field with the aid of a probe which is the averaging volume V. It is obvious that using probes with different geometries one obtains different descriptions of the same physical flow field. This dependence on V is also manifested in the governing equations for the averaged flow descriptors. The equations consist of a system of partial differential equations with appropriate initial and boundary conditions, and a closure of the system by a set of constitutive equations. These equations, notably the initial and boundary conditions and the constitutive equations, are shown easily to depend on V (Celmiņš & Schmitt 1984; Dobran 1984). As a consequence, the solution of the governing equations also depends on the size and shape of V, which is in agreement with the intuitive interpretation of V as a probe of the flow field. A reasonable discussion of a two-phase flow field in terms of space-averaged quantities, therefore, requires a specification of the averaging volume for which the results are assumed to hold. Most authors avoid the specification of V by assuming tacitly or explicitly that the scale of V is much smaller than the scales of salient flow structures and much larger than an average distance between particles. Explicit statements of this condition on V are given, for example, by Drew (1971), Gough (1974), Immich (1980) and Celmiņš & Schmitt (1984). If the scale of V is within this range then one can expect little dependence of the averaged flow field on V. However, there are flows of practical interest for which an averaging volume with the quoted properties does not exist. One example is a boundary layer flow with particles which have diameters that are not much smaller than the boundary layer thickness. Another example is a flow through a tube where the tube diameter is not orders of magnitude larger than an average distance between particles. In order to describe salient flow features in these examples, the averaging volume must be chosen so small that it is not much larger than an average distance between particles. In this regime, one can expect a strong dependence of the averaged flow (and its governing equations) on the size and shape of V and, therefore, a discussion of average flow equations and their solution is meaningless if V is not specified. The above-mentioned examples and similar flows should be analyzed in terms of quantities other than volume-averaged descriptors.

The purpose of the present paper is to find a quantitative bound for the validity of the usual volume-averaged flow equations and flow descriptors. The problem is approached by considering the possibly simplest two-phase medium: a flow with uniform particle distribution, sampled with V at places sufficiently remote from flow boundaries. If V is large, then one obtains for the gas volume ratio α with V practically the same value for any position of V. This value of α is close to the limit value $\bar{\alpha}$ for infinitely large V in an unrestricted medium. Hence, using a large V, one can reasonably describe the mixture by a single constant $\bar{\alpha}$. If, on the other hand, V is very small, e.g. smaller than the particles, then the averaged flow is not uniform at all: at the location of a particle the gas volume ratio α drops to zero, and between particles the value of α is close to unity (Such a detailed description of a two-phase flow should not be analyzed in terms of space-averaged two-phase equations.) These examples show that the degree of apparent uniformity of the particle distribution depends on the size of the probe V. Hence, one can ask for a lower bound of V above which a given particle distribution appears to be reasonably uniform. The bound depends on the particle sizes, the average distance between particles and on the meaning of "reasonably uniform". The latter concept we quantitate by a tolerance level for the difference between the actual α for a finite V and the limit value $\bar{\alpha}$ for infinitely large V. One can show that the amplitudes of the deviations of other flow descriptors (such as average velocity, density etc.) from their corresponding limit values are proportional to the difference $|\alpha - \bar{\alpha}|$ (Celmiņš 1984). Therefore, a tolerance expressed in terms of the deviations $\alpha - \bar{\alpha}$ will in most cases provide an adequate order of magnitude bound for the size of V.

We note in passing that the averaging volume should not be confused with the so-called control volume which is used in the derivation of conservation equations. The latter can be reduced to infinitesimal size, for instance, in order to obtain conservation equations in differential form. In contrast, the averaging volume is finite and fixed (at least within finite limits) for any given flow description. It cannot be freely changed without changing the governing equations, particularly the constitutive equations. In particular, it cannot be reduced to zero, because the concept of averaging always implies a finite averaging volume, much larger than the particles in the flow.

The calculation of extreme undulations of the gas volume fraction α in a uniform gas–particle mixture is described in section 2. Sample calculations with different uniform particle distributions and different sizes of V can be used to obtain an empirical relation between the size of V and the amplitudes of undulations of α. The derivation of such a relation is outlined in section 3. Section 4 contains a discussion of the results and section 5 is a summary.

2. UNDULATIONS OF α IN UNIFORM PARTICLE AGGREGATES

Let the averaging volume V be a sphere with radius R, the particles be spheres with radii s and let the particle aggregate consist of m particles and occupy the volume W. Then the average gas volume fraction of the two-phase medium contained in W is

$$\bar{\alpha} = 1 - \frac{4}{3}\frac{\pi s^2 m}{W}. \quad [1]$$

One obtains a measure for the mean distance between particle centers by assigning to each particle the volume fraction W/m and representing this volume fraction as a virtual sphere. The diameter L_m of the virtual sphere is given by the formula

$$L_m = 2s(1 - \bar{\alpha})^{-1/3}, \quad [2]$$

and it may be defined as the mean distance between particle centers. If the diameter of the averaging sphere V is much larger the L_m, then a good estimate for the number of particles contained in V is, in terms of L_m, given by

$$n = \left(\frac{2R}{L_m}\right)^3, \quad [3]$$

and the gas volume fraction α within V is very close to $\bar{\alpha}$ for any position $\mathbf{X}$ of the center of V.

For arbitrary V, the dependence of α on the position and size of V can be expressed by a function of the following type:

$$\alpha = f\left(\frac{\mathbf{X}}{L_m}, \frac{R}{L_m}, \bar{\alpha}\right). \qquad [4]$$

The particular form of f depends on dimensionless parameters which describe the geometry of the particle arrangement. For a given aggregate and averaging sphere, $\bar{\alpha}$ and R/L_m are constants, and α depends only on the position vector of the averaging sphere, i.e. on the first argument in [4]. As V is moved along a trajectory through the aggregate, α undulates about its limit value $\bar{\alpha}$. Our goal is to obtain estimates for the bounds of the amplitudes of these unwanted undulations for arbitrary trajectories of V through aggregates with uniform particle distributions.

The particle aggregates should be chosen for the present purpose such that one reasonably would qualify them as representing uniform distributions without specifying the size of the averaging volume. (As pointed out above, *any* aggregate can be considered as uniform if V is sufficiently large.) This means that the aggregate should not have obvious particle clusters, periodic or random. It seems that the best examples of such aggregates are lattice points in regular lattices. Four examples of such lattices are defined in the appendix, and more examples can be easily constructed. The simplest example in the appendix is the square cyclinder lattice (cubic lattice). It represents an aggregate with a very low sphere packing density. The next example is a triangular cylinder lattice with an intermediate sphere packing density, and the two remaining examples (leap-frog square and leap-frog triangular lattices) are both lattices with the highest possible sphere packing densities. The orientations and lateral positions of the lattices are arbitrary. All four lattices clearly represent aggregates with uniform particle distributions in the outlined sense.

We have not calculated the undulations of α in any example of a "uniform and random" particle distribution because the concepts of spatial uniformity and spatial randomness are contradictory. A randomness in the spatial distribution implies (random) clusters of particle arrangements, i.e. deviations from uniformity. Therefore, for a given $\bar{\alpha}$ the undulations of α in a "random" particle aggregate necessarily are larger than in a "uniform" aggregate in the form of a regular lattice.

The calculation of α in terms of the arguments in [4] is most easily arranged for fixed positions $\mathbf{X}$ of V, and a variable radius R of V. Figure 1 shows two examples of such a calculation, corresponding to two positions of the averaging sphere in a square cylinder lattice array of particles with $\bar{\alpha} = 0.9$. One position of the averaging sphere is the point of origin which is a lattice point and occupied by a particle. At that point, $\alpha \equiv 0$ for $R \leqslant s$, and α starts to increase only as R becomes larger than s. The other curve in figure 1 is for the position $\mathbf{X}/L_m = (0.4, 0.2, 0.0)$, which is a point occupied by gas. The corresponding curve $\alpha(R/L_m)$ starts at $R = 0$, and decreases only as the expanding sphere encounters the first particle. As R further increases, both curves undulate with decreasing amplitude about the limit value $\bar{\alpha} = 0.9$. The differences in amplitude and wavelength of the undulations between the two examples indicate the dependence of α on the position vector $\mathbf{X}/L_m$. The two sample curves belong to a family of curves with three parameters, namely the three components of $\mathbf{X}/L_m$. We are interested in the extreme deviations of α from $\bar{\alpha}$ for arbitrary positions of the averaging sphere, i.e. for arbitrary positions and orientations of the lattice with respect to a fixed sphere. These extremes can be obtained by calculating the envelopes of the three-parameter curve family. The result is shown in figure 2. The irregular shape of the

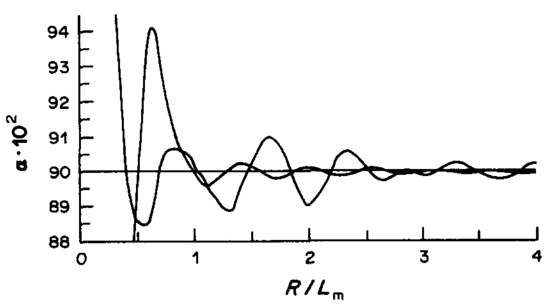

Figure 1. Gas volume fraction dependence on averaging sphere radius: square cylinder lattice, $\bar{\alpha} = 0.9$.

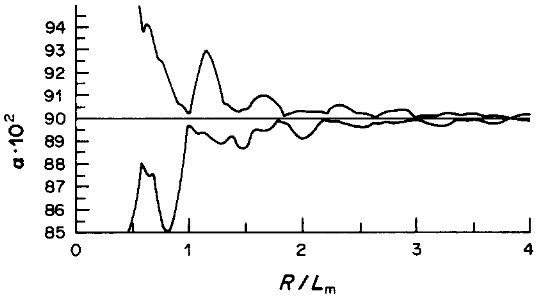

Figure 2. Envelopes of gas volume fraction curves: square cylinder lattice, $\bar{\alpha} = 0.9$.

envelopes is somewhat unexpected, considering the regularity of the particle aggregate. Such an irregularity of the envelopes was found to be typical for all four lattices. Examples of individual curves and envelopes for a leap-frog triangular lattice with $\bar{\alpha} = 0.5$ are shown in figures 3 and 4, respectively. Further examples were given previously (Celmiņš 1984). In all calculated cases (with various values of $\bar{\alpha}$ and for the four lattices considered) one finds that the extreme deviations of the envelopes from $\bar{\alpha}$ are aproximately proportional to a negative power of R/L_m, and that the wavelength of the undulations of $\alpha - \bar{\alpha}$ is of the order L_m. These trends are investigated in more detail in the next section.

3. UNDULATION BOUNDS AND TOLERANCE CONDITIONS FOR R/L_m

One obtains an overview about the trends of the deviations $\Delta\bar{\alpha} = \alpha - \bar{\alpha}$ by plotting the extreme values of the envelopes of $\alpha(R/L_m)$ vs the nondimensionalized radius R/L_m of the averaging sphere. Figure 5 shows such a plot for $\bar{\alpha} = 0.5$. The different symbols in the plot signify different lattices and positive or negative deviations. The scatter of the points indicate that none of the four lattices consistently produces larger or smaller deviations, and that the extreme positive and negative deviations are of the same order of magnitude. The straight line in figure 5 represents an estimated upper bound for the deviations. By comparing such plots for different values of $\bar{\alpha}$ one finds the following empirical equation for the upper bound:

$$|\Delta\bar{\alpha}| = 0.5\bar{\alpha}^2(1-\bar{\alpha})\left(\frac{R}{L_m}\right)^{-2}. \qquad [5]$$

Now we discuss the validity of this equation.

The bound estimate [5] is based on calculations within the range $1.0 \leqslant R/L_m \leqslant 4.0$ for $\bar{\alpha} = 0.5$, 0.667 and 0.9, and for the four lattices described in the appendix. Because the lattices have quite different symmetries and, because they represent aggregates with maximal packing densities ranging from $\bar{\alpha}_{min} = 0.260$ to 0.476, one can assume that the results are valid for all reasonably uniform particle arrangements. The bound [5] cannot be improved in the sense that calculations of additional examples of uniform aggregates might only make the bound larger but we consider this to be unlikely. The limitation of the calculations to $\bar{\alpha} \geqslant 0.5$ was motivated by the observation that the lowest possible value of $\bar{\alpha}$ for the square cylinder lattice is 0.476. Therefore, one can obtain

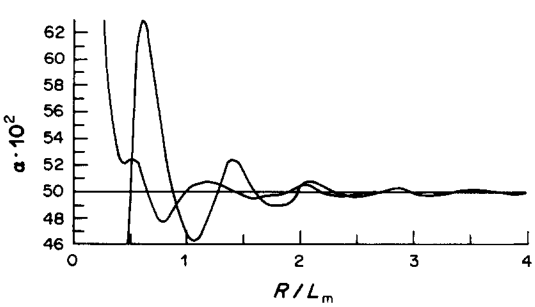

Figure 3. Gas volume fraction dependence on averaging sphere radius: leap-frog triangular lattice, $\bar{\alpha} = 0.5$.

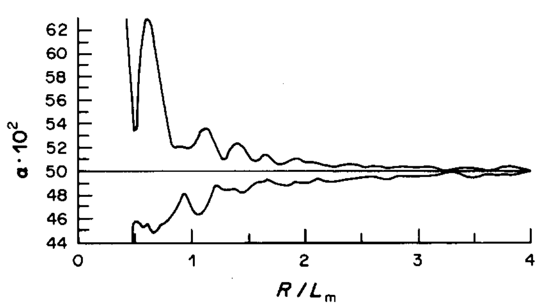

Figure 4. Envelopes of gas volume fraction curves: leap-frog triangular lattice, $\bar{\alpha} = 0.5$.

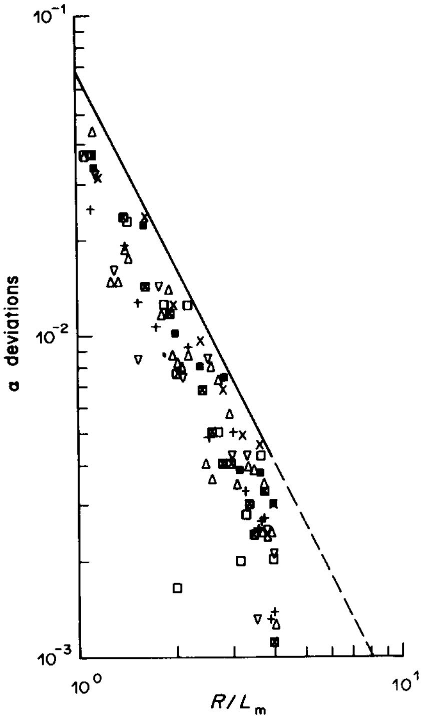

Figure 5. Extreme deviations of gas volume fraction. Combined plot for four lattices, and positive and negative deviations, $\bar{\alpha} = 0.5$.

an $\bar{\alpha} \ll 0.5$ only in special dense packings of spheres, whereas $\bar{\alpha} \geqslant 0.5$ can be realized with any reasonably uniform particle aggregate. The upper limit of $\bar{\alpha}$ is unity (gas only). Formula [5] produces for this limit the correct value $\Delta\bar{\alpha} = 0$. Therefore, one can expect that it also adequately interpolates between $\bar{\alpha} = 1.0$ and the calculated value for $\bar{\alpha} = 0.9$, so that it is a reasonable estimate of the bound for the whole range $0.5 \leqslant \bar{\alpha} \leqslant 1.0$. The calculations were limited to $R/L_{\mathrm{m}} \geqslant 1$, because for smaller R/L_{m} the maximum amplitudes of the undulations become excessively large. At $R/L_{\mathrm{m}} = 4.0$, the amplitudes are of the order 10^{-3} which can be assumed sufficiently small for most applications.

Equation [5] may be solved for R/L_{m} and used as a condition for the minimum size of the averaging volume in terms of a prescribed tolerable deviation $|\Delta\bar{\alpha}|_{\mathrm{tol}}$. Then the equation produces the following condition:

$$\frac{R}{L_{\mathrm{m}}} > \bar{\alpha}\left[\frac{0.5(1-\bar{\alpha})}{|\Delta\bar{\alpha}|_{\mathrm{tol}}}\right]^{1/2}. \qquad [6]$$

If R/L_{m} satisfies this condition $\bar{\alpha}$, then the undulations of α are less than $|\Delta\bar{\alpha}|_{\mathrm{tol}}$.

One may choose a constant value for $|\Delta\bar{\alpha}|_{\mathrm{tol}}$ if $\bar{\alpha}$ is known to be less than unity throughout the flow. However, if $\bar{\alpha}$ is approaching unity in some parts of the flow, then a more reasonable expression for $|\Delta\bar{\alpha}|_{\mathrm{tol}}$ is a function proportional to $1 - \bar{\alpha}$. For instance, one may define the tolerance level as follows:

$$|\Delta\bar{\alpha}|_{\mathrm{tol}} = \begin{cases} |\Delta\bar{\alpha}|_{t} & \text{if} \quad \bar{\alpha} \leqslant \bar{\alpha}_{t} \\ \dfrac{1-\bar{\alpha}}{1-\bar{\alpha}_{t}}|\Delta\bar{\alpha}|_{t} & \text{if} \quad \bar{\alpha}_{t} < \bar{\alpha}, \end{cases} \qquad [7]$$

with proper values of $|\Delta\bar{\alpha}|_{t}$ and α_{t}. The corresponding form of [6] is

$$\frac{R}{L_{\mathrm{m}}} > \bar{\alpha}[\max\{1-\bar{\alpha}, 1-\bar{\alpha}_{t}\}]^{1/2}\left(\frac{1}{2|\Delta\bar{\alpha}|_{t}}\right)^{1/2}. \qquad [8]$$

We assume for simplicity that $\bar{\alpha}_t > 0.85$ and $|\Delta\bar{\alpha}|_t < 0.15$, and observe that under these restrictions the factor of $|\Delta\bar{\alpha}|_t^{1/2}$ in [8] has a maximum at $\bar{\alpha} = 2/3$, a minimum at $\bar{\alpha} = \alpha_t$ and is linearly increasing with $\bar{\alpha}$ between $\bar{\alpha} = \bar{\alpha}_t$ and $\bar{\alpha} = 1$. Hence, if $\bar{\alpha}$ is not known, then [8] should be used with $\bar{\alpha} = 2/3$, because this gives the largest value of the r.h.s. If $\bar{\alpha}$ is known, for instance, to be larger than $\bar{\alpha}_t$, then [8] should be used with $\bar{\alpha} = 1$.

Figure 6 is a graphical display of [8] for $\alpha_t = 0.9$. It shows, for instance, that for $|\Delta\bar{\alpha}|_t = 0.01$ the averaging sphere's radius R should be larger than $2.7L_m$, if $\bar{\alpha}$ is not known, and larger than $2.2L_m$ if $\bar{\alpha}$ is known to be larger than 0.867. The solid lines in the figure indicate the domain in which sample calculations have been done. Extrapolations are indicated by dashed lines.

Relations [6] or [8] can also be expressed in terms of the minimum required number of particles in the averaging sphere. Using [3] and [8] one obtains for the number n the following condition:

$$n > \bar{\alpha}^3[\max\{1-\bar{\alpha}, 1-\alpha_t\}]^{3/2}\left(\frac{1}{2|\Delta\bar{\alpha}|_t}\right)^{3/2}. \quad [9]$$

Condition [9] is displayed in figure 7 for $\bar{\alpha} = 0.9$. It shows that for $|\Delta\bar{\alpha}|_t = 0.01$ the minimum number of particles in the averaging sphere is between 65 and 160, depending on $\bar{\alpha}$. The condition has been tested by calculations with n between 8 and 512, as indicated by the solid parts of the lines in the figure.

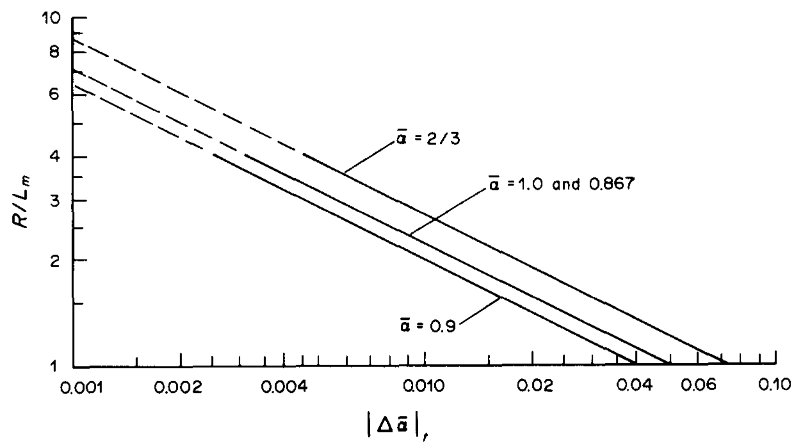

Figure 6. Minimum radius of averaging sphere for given tolerance and $\bar{\alpha}_t = 0.9$.

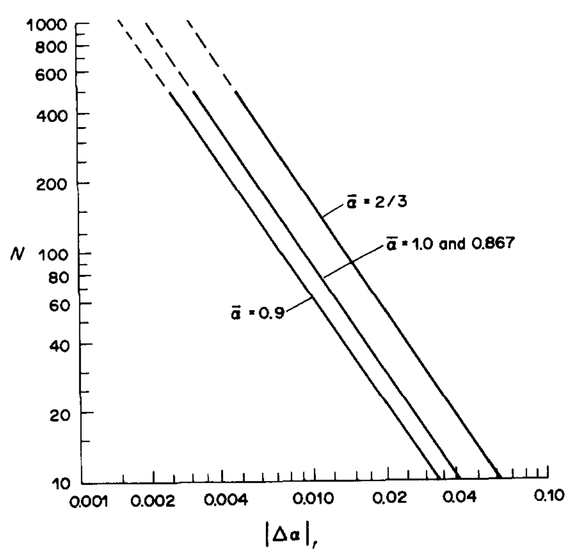

Figure 7. Minimum number of particles in an averaging volume for given tolerance and $\bar{\alpha}_t = 0.9$.

4. DISCUSSION OF THE RESULT

In the preceding section, a relation was derived between the extreme amplitudes of undulations of α in a medium with uniform particle distribution, and a lower bound for the radius R of the averaging volume V. The important result is that the bound for R is finite and typically equals 2 or more mean distances between particle centers.

Let ϕ be another flow descriptor, e.g. the gas velocity. If ϕ is constant then its average is also constant and equals ϕ, regardless of the size of V. Undulations of α do not affect this average. However, if ϕ is not constant, then its average undulates with an amplitude which is proportional to the amplitude $|\Delta\bar{\alpha}|$ of the undulations of α (Celmiņš 1984). For instance, if ϕ has a constant gradient and changes by $\delta\phi$ along a diameter of V in the gradient direction, then the amplitude $|\Delta\phi|$ of the undulations of the average ϕ is given by

$$|\Delta\phi| \approx \frac{|\delta\phi\,\|\,\Delta\bar{\alpha}|}{3\alpha} \qquad [10]$$

(An example of a transient two-phase flow with constant gradient is the flow in a gun. At any fixed time, the gas and particle velocities have a constant gradient practically over the whole length of the tube between the breech and projectile.) Another example of an undulating flow descriptor is a ϕ which equals a constant value ϕ_0 throughout the gas except for boundary regions around each particle. Let $|d\phi|$ be the magnitude of the difference between the average ϕ and ϕ_0. Then the undulations of the average ϕ have an amplitude $\Delta\phi$ which can be estimated by

$$|\Delta\phi| \approx \frac{|d\phi\,\|\,\Delta\bar{\alpha}|}{[\alpha(1-\alpha)]}. \qquad [11]$$

Estimates [10] and [11] may be used in combination with the formulas for the bound of R in section 3 to obtain a specific value for the lower bound in applications where a particular descriptor ϕ is of principal interest.

The existence of a finite lower bound of the averaging sphere's radius has consequences for the representation and computation of two-phase flows. Let R_{min} be the value of the bound, i.e. we assume that the flow field is described using quantities which are averaged over a V with a radius $R \geqslant R_{min}$. Then any flow structures with extensions less than R_{min} will be reduced in amplitude and stretched out to a size of $2R_{min}$ or larger. Consequently, a complete structure and accurate representation of the field can be made on a computing mesh with mesh size $R_{min}/2$. Any mesh with mesh refinement can be done by interpolation in such a net, and a finer net does not provide a more accurate description of the flow. The same applies to flow measurements and flow calculations. Local flow measurements should be averaged over V and presented in the described coarse mesh. Calculations of the flow field, e.g. by numerically solving the governing equations, should be performed with a mesh size of the order $R_{min}/2$ so that flow structures with extensions larger than, say, R_{min} can be captured. Any refinement of the computing mesh below $R_{min}/2$ has the effect of interpolation. If flow structures extending less than R_{min} appear in the solution (using a fine mesh or an analytical solution) then they should be interpreted as numerical artifacts or noise, because they cannot be interpreted as average flow properties. A likely cause of such structures is the failure to use constitutive equations and boundary conditions appropriate for the size and shape of the averaging volume.

When averaged descriptors are used to represent transient flows in which α approaches unity in some parts of the flow field, then one should distinguish between the possible causes for α approaching unity. If the reason for the disappearance of the particles is the reduction of their sizes (for instance, by combustion), then L_m does not change and consequently R_{min} is not affected. Therefore, such regions with $\alpha \approx 1$ can be represented by the same average descriptors as the rest of the field. If, however, α approaches unity because particles diffuse from the mixed-phase region, then L_m increases. Consequently, R_{min} also increases, i.e. such a flow field has to be averaged with a larger V to guarantee the tolerance level of the undulations of α or a decrease of the accuracy of the representation must be taken into account. A better approach in the case of diffusing particles is to model the rarified parts of the aggregate by some other method than averaging, for instance, by individually tracing the diffused particles.

As an example, we consider an interior ballistics two-phase flow where the particulate phase consists of propellant grains. Let the chamber volume be W_C and the volume of the barrel be W_B. At the time when the projectile exists the barrel, the volume available to the gas–particle mixture is $W_C + W_B$. Let the number of grains be m. Then the mean distance between particle centers at the beginning of the firing cycle is equal to

$$L_{m0} = \left(\frac{6}{\pi}\frac{W_C}{m}\right)^{1/3}. \quad [12]$$

At muzzle time, the mean distance equals

$$L_m = \left(\frac{6}{\pi}\frac{W_C + W_B}{m}\right)^{1/3}. \quad [13]$$

Hence, the mean distance increases during the firing cycle by the factor

$$\frac{L_m}{L_{m0}} = \left(1 + \frac{W_B}{W_C}\right)^{1/3}. \quad [14]$$

Let the initial gas volume fraction in the gun be α_0, and the initial particle radius be s_0. Then

$$L_{m0} = 2s_0(1 - \alpha_0)^{-1/3}$$

and

$$L_m = 2s_0(1 - \alpha_0)^{-1/3}\left(1 + \frac{W_B}{W_C}\right)^{1/3}. \quad [15]$$

Typical for interior ballistics is an α_0 between 0.4 and 0.6, and $W_B/W_C \approx 10$. Therefore, the maximum of $L_m/(2s_0)$ is, for a typical gun, between 2.64 and 3.02. From figure 1 one finds the condition $R \geqslant 2.7L_m$ for $|\Delta\alpha|_{tol} = 0.01$. Hence, in order to represent the entire transient firing cycle by averaged descriptors, one has to use an averaging sphere with a radius of about 8 initial particle diameters. For most guns, the diameter of such an averaging sphere is of the same order as the caliber of the tube. This means that in typical gun the radial flow structure cannot be represented and discussed in terms of space-averaged descriptors. Space averaging in such cases is only meaningful if it is done over cross-sectional segments of the tube, i.e.for the calculation of the core flow.

5. SUMMARY AND CONCLUSIONS

The description of two-phase flow in terms of space averages always implies a finite averaging volume. The size of the volume affects the flow description. This becomes evident by considering extreme volume sizes. If the averaging volume is very large, then all flow structures are smoothed out and one obtains a flow with constant properties. If the volume is very small, then each particle is visible in the averages and at the limit one has the local flow description. Hence there are upper and lower bounds for a reasonable size of the averaging volume. The bounds depend on the problem which one wants to investigate, and one hopes that the lower bound does not exceed the upper bound, so that an appropriate averaging volume can be identified. However, there are examples of important two-phase flows for which such a volume cannot be identified and which, therefore, cannot be analyzed in terms of volume-averaged descriptors.

An upper bound for the averaging volume is given simply by the dimensions of flow structures which one wants to investigate. The dimensions of the averaging volume should be sufficiently small so that the salient flow structures are not smoothed out.

A lower bound is more difficult to specify. The present paper provides an empirical formula for this specification which gives the bound in terms of tolerable undulations of the gas volume ratio in particle aggregates with uniform distribution. One finds that this bound is quite large. For instance, in terms of particle numbers, a reasonable averaging volume should contain at least 60–150 particles. Because of this finite lower bound, one should be careful when discussing details of flow structures, like boundary layers or radial profiles of tube flows. Such discussions are only meaningful if one can specify the averaging volume, or the range of volumes, for which the results hold. One can show, for instance, that for two-phase flows through tubes the radial profiles of average descriptors are only meaningful if the distances between particles are at least an order of magnitude smaller than the tube radius. Similar restriction hold for boundary layer flows.

The results of the present paper permit one to determine explicitly, for given particle sizes and distributions, what kind of flow structures can be reasonably discussed in terms of space-averaged descriptors.

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APPENDIX

Lattices

We describe in this appendix the four lattices which were used to define particle positions for calculation of the gas volume fractions.

1. Square cylinder lattice

We construct the lattice by first arranging the particles in a square mesh with the mesh constant L in the x, y-plane, and then translating the mesh by multiples of L in the z-direction. Each square thereby generates a square cylinder. In this lattice, each particle has 6 neighbors at the distance L.

2. Triangular cylinder lattice

The lattice is constructed by first arranging the particles in the x, y-plane in an equilateral triangle mesh with the mesh constant L, and then translating the mesh in the z-direction by multiples of L. Each triangle thereby generates a triangular cylinder. The number of neighbor particles at distance L for this lattice is 8.

3. Leap-frog square lattice.

This lattice is constructed by starting with a square mesh in the x, y-plane, with the lattice constant L and the sides of the squares parallel to the axes. Then, the mesh is translated by multiples of $L/\sqrt{2}$ in the z-direction and by multiples of $L/2$ in the x- and y-directions. Thus, the pattern is translated in a leap-frog manner from one z-plane to the next. Each particle in this lattice has 12 neighbor particles at distance L.

4. Leap-frog triangular lattice

The lattice is constructed by first arranging the particles in an equilateral triangular mesh in the x, y-plane, with the mesh constant L and one side of the triangle parallel to the x-axis. Then the mesh is translated in the z-direction by multiples of $L\sqrt{2/3}$, and in the y-direction alternatively by $\pm L\sqrt{3}$. Thus, the triangular mesh is shifted in a leap-frog manner from one z-plane to the next. The number of neighbors at distance L from any particle in this lattice is 12.

Gas Volume Fraction

The minimum value of the gas volume fraction (closest packing of spheres) is obtained in the four lattices by setting the particle radius $s = L/2$. The numerical values of $\bar{\alpha}_{\min}$ are as follows:

Square cylinder $\quad \bar{\alpha}_{\min} = 1 - \dfrac{\pi}{6} = 0.476.$

Triangular cylinder $\quad \bar{\alpha}_{\min} = 1 - \dfrac{\pi}{3\sqrt{3}} = 0.395.$

Leap-frog lattices $\quad \bar{\alpha}_{\min} = 1 - \dfrac{\pi}{3\sqrt{2}} = 0.260.$

Both leap-frog lattices are arrangements with closest packing of spheres in three-dimensions.